Finite Math - J-term 2019

Lecture Notes - 1/22/2019

Homework

- Section 5.3 - 1, 2, 5, 6, 9, 17, 19, 21, 23, 25, 27, 49, 50, 53

Section 5.3 - Linear Programming in Two Dimensions: A Geometric Approach

Theorem 1 (Fundamental Theorem of Linear Programming)**.** *If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or more of the corner points of the feasible region.*

Theorem 2 (Existence of Optimal Solutions)**.**

(A) *If the feasible region for a linear programming problem is bounded, then both the maximum value and the minimum value of the objective function always exist.*

(B) *If the feasible region is unbounded and the coefficients of the objective function are positive, then the minimum value of the objective function exists, but the maximum value does not.*

(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

Geometric Method for Solving Linear Programming Problems.

Procedure (Geometric Method for Solving a Linear Programming Problem with Two Decision Variables)**.**

(1) *Graph the feasible region. Then, if an optimal solution exists according to Theorem 2, find the coordinates of each corner point.*

(2) *Construct a* corner point table *listing the value of the objective function at each corner point.*

(3) *Determine the optimal solution(s) from the table in Step 2 (smallest=minimum, largest=maximum).*

(4) *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Example 1. *Maximize and minimize* $z = 3x + y$ *subject to the inequalities*

$$\begin{aligned} 2x + y &\leq 20 \\ 10x + y &\geq 36 \\ 2x + 5y &\geq 36 \\ x, y &\geq 0 \end{aligned}$$

Solution.

Example 2. *Maximize and minimize* $z = 2x + 3y$ *subject to*

$$\begin{aligned} 2x + y &\geq 10 \\ x + 2y &\geq 8 \\ x, y &\geq 0 \end{aligned}$$

Example 3. *Maximize and minimize* $P = 3x + 5y$ *subject to*

$$\begin{aligned} x + 2y &\leq 6 \\ x + y &\leq 4 \\ 2x + 3y &\geq 12 \\ x, y &\geq 0 \end{aligned}$$

Applications.

Example 4. *An electronics firm manufactures two types of personal computers–a desktop model and a laptop model. The production of a desktop requires a capital expenditure of* \$400 *and* 40 *hours of labor. The production of a laptop requires a capital expenditure of* \$250 *and* 30 *hours of labor. The firm has* \$20,000 *capital and* 2,160 *labor-hours available for production of standard and portable computers.*

(a) *What is the maximum number of computers the company is capable of producing?*

(b) *If each desktop contributes a profit of* \$320 *and each laptop contributes a profit of* \$220*, how much profit will the company make by producing the maximum number of computers?*

(c) *Does producing as many computers as possible produce the highest profit? If not, what is the highest profit and how many of each computer should be made in that case?*

Example 5. *A fruit grower can use two types of fertilizer in his orange grove, brand A and brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chloride in a bag of each brand are given in the table. Tests indicate that the grove needs at least* 1,000 *pounds of phosphoric acid and at most* 400 *pounds of chloride.*

	Brand A	*Brand B*
Nitrogen	8	3
Phosphoric Acid	4	4
Chloride	2	1

(a) *If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*

(b) *If the grower wants to minimize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?*